# Complete Eigenvalue Spectrum for the Nucleation in a Ferromagnetic Prolate Spheroid

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Curling, coherent rotation, and an appropriately defined buckling are proved to be the only three modes which might yield numerically smallest nucleation field in ellipsoids of revolution (both prolate and oblate), all the other possible eigenmodes yielding more negative nucleation fields. A lower bound for the buckling mode in a prolate spheroid is calculated by neglecting the transverse magnetostatic self-energy. In contradistinction to the limiting case of an infinite circular cylinder, in which coherent rotation never takes place, it is found that for prolate spheroid of any finite elongation, coherent rotation is the lowest eigenmode for small enough radii. Upper and lower bounds are given for the critical radius under which coherent rotation takes place, as a function of the elongation. Buckling is seen to take place at most in a rather small region of sizes between curling and coherent rotation.

## 1. INTRODUCTION

MAGNETIZATION reversal in a previously saturated ferromagnetic particle was shown by Brown[1] to follow one of the eigenfunctions of a set of linear partial differential equations with boundary conditions. The reversal starts at a certain value of the applied field, the so-called "nucleation field," which is the least negative eigenvalue of this set of equations, and the reversal should follow the eigenmode associated with this eigenvalue.

For the two limits of a prolate spheroid, namely, the sphere and the infinite circular cylinder, Brown[1] could guess two solutions, coherent rotation and curling. For an infinite cylinder, Frei *et al.*[2] suggested another mode, which they called "buckling." They showed that the magnetization could reverse by this mode more easily than by coherent rotation. The mode they studied was not an eigenmode of Brown's equation, which meant the nucleation field it yielded could not be a minimum, and a numerically smaller eigenvalue should have existed. However, a rigorous calculation[3] showed the eigenvalue was only about 1% smaller than the value yielded by the buckling approximation, and the eigenmode highly resembled the assumed function. This *exact* eigenmode will, therefore, be referred to here as the buckling mode.

For an infinite cylinder, the whole eigenvalue spectrum of Brown's equations has been studied,[3] and it was found that only curling and buckling modes could yield numerically smallest eigenvalues. The buckling yields the numerically smallest eigenvalue when the cylinder radius is smaller than about 1.1 $A^{1/2}I_s^{-1}$ (where $A$ is the exchange constant, $I_s$ is the saturation magnetization), while curling yields the lowest eigenvalue for a radius larger than this value. In the other extreme case of a sphere, it has been proved[4] that the lowest eigenmodes are coherent rotation and curling, where the former is the lowest for a radius smaller than about 1.4 $A^{1/2}I_s^{-1}$, while the latter is the smallest above this radius.

The coherent rotation is an eigenmode for the general ellipsoid, in particular, for the prolate spheroid discussed here.[5] The curling is also an eigenmode of an ellipsoid of revolution, and its eigenvalue has been calculated for the general prolate spheroid, as a function of elongation.[4] In the Sec. 2, we shall show that just another eigenmode, analogous to the buckling, should be added to the picture, and all the other possible modes yield eigenvalues which are more negative than these three, so that they can never be reached. In Sec. 3 this buckling mode will be treated approximately. This approximation will be seen to yield reasonably close upper and lower bounds for the critical size for coherent rotation ("single domain" behavior), and a lower bound for curling. Finally, in Sec. 4 some aspects of the analogous problem in oblate spheroid will be outlined.

## 2. CURLING AND HIGHER MODES

Consider a prolate spheroid made of a ferromagnetic material, and let its axis of symmetry be chosen as the $z$ axis. Let the external field $H$ be applied along the $z$ axis, which is also assumed to be an easy axis for magnetocrystalline anisotropy energy, which can be either cubic or unidirectional, with a coefficient $K$. Let the direction cosines of the transverse magnetization, in a *cylindrical*-coordinate system $r$, $\varphi$, $z$, be $\alpha_r$ and $\alpha_\varphi$; and let $U$ be the potential of surface and volume charges due to *transverse* magnetization. It is then seen, by following the derivation of the equations,[3] that in cylindrical coordinates the Brown equations are essentially the same as for the infinite cylinder, except for including the demagnetizing field due to surface charges in the saturated magnetization state along $z$. The nucleation field, $H_n$, is thus the least negative eigenvalue, $H$, of the

[1] W. F. Brown, Jr., Phys. Rev. **105**, 1479 (1957). For a more complete list of references, see A. Aharoni, Rev. Mod. Phys. **34**, 227 (1962).
[2] E. H. Frei, S. Shtrikman, and D. Treves, Phys. Rev., **106**, 446 (1957).
[3] A. Aharoni and S. Shtrikman, Phys. Rev. **109**, 1522 (1958).
[4] A. Aharoni, Suppl. J. Appl. Phys. **30**, 70 (1959).
[5] W. F. Brown, Jr., *Magnetostatic Principles in Ferromagnetism* (North-Holland Publishing Company, Amsterdam, 1962), Chap. 6.

following set of equations:

$$(\nabla'^2 - t^{-2} - \pi S^2 h)\alpha_r - 2t^{-2}\partial\alpha_\varphi/\partial\varphi - \pi S\partial u/\partial t = 0, \quad (1a)$$

$$(\nabla'^2 - t^{-2} - \pi S^2 h)\alpha_\varphi + 2t^{-2}\partial\alpha_r/\partial\varphi - \pi S t^{-1}\partial u/\partial\varphi = 0, \quad (1b)$$

$$\nabla'^2 u = 2S[\partial\alpha_r/\partial t + t^{-1}(\alpha_r - \partial\alpha_\varphi/\partial\varphi)], \quad (1c)$$

inside the spheroid. Outside, the potential fulfills the Laplace equation

$$\nabla'^2 u_{\text{out}} = 0. \quad (2)$$

On the boundary,

$$\partial\alpha_r/\partial n = \partial\alpha_\varphi/\partial n = 0, \quad (3a)$$

$$u_{\text{in}} = u_{\text{out}}, \quad (3b)$$

$$2S\alpha_n = \partial u_{\text{in}}/\partial n - \partial u_{\text{out}}/\partial n. \quad (3c)$$

Here

$$\nabla'^2 = \frac{\partial^2}{\partial t^2} + \frac{1}{t}\frac{\partial}{\partial t} + \frac{1}{t^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial p^2}, \quad (4)$$

where

$$t = r/R, \quad p = z/R, \quad S = RI_s A^{-1/2},$$

$$h = \frac{H}{2\pi I_s} + \frac{K}{I_s^2} - \frac{N}{2\pi}, \quad u = \frac{U}{2\pi A^{1/2}}. \quad (5)$$

$R$ is the radius of the spheroid in a direction perpendicular to $z$, $n$ is the normal to its surface, and $N$ is the demagnetizing factor along $z$.

Because of the cylindrical symmetry, the $\varphi$ dependence can be readily separated by writing

$$\alpha_r = A_r(t,p)\cos(m\varphi - \varphi_0), \quad (6a)$$

$$\alpha_\varphi = A_\varphi(t,p)\sin(m\varphi - \varphi_0), \quad (6b)$$

$$u = V(t,p)\cos(m\varphi - \varphi_0), \quad (6c)$$

where $m$ is an integer, so that the $\varphi$ dependence has the necessary periodicity of $2\pi$. It is seen that by using (6), the variable $\varphi$ can be eliminated both from the equations and the boundary conditions. They can, therefore, be solved separately for every integral value of $m$.

If $m=0$, the equation for $A_\varphi$ is separated from the rest and is

$$\left[\frac{\partial^2}{\partial t^2} + \frac{1}{t}\frac{\partial}{\partial t} - \frac{1}{t^2} + \frac{\partial^2}{\partial p^2} - \pi S^2 h\right] A_\varphi = 0 \quad (7a)$$

with the boundary condition

$$\partial A_\varphi/\partial n = 0. \quad (7b)$$

This is the equation for the curling mode, the eigenvalues for which have already been computed for the case of a prolate spheroid,[4] and are reproduced in Fig. 1. The other equations and boundary conditions, involving $A_r$ and $V$ need to be treated, since it is readily seen that by dropping the positive term of transverse magnetostatic self energy, these equations reduce to (7). Therefore, if this term is retained in the energy, the nucleation field for this other mode cannot be less negative than the curling. One can, therefore, conclude that for $m=0$ the lowest eigenvalue is obtained for $u=\alpha_r=0$ and is the eigenvalue of (7).

For $m \geqslant 2$, when the transverse magnetostatic self-energy is again dropped and (6) is substituted in (1) and (3), one obtains the following two separate equations:

$$\left[\frac{\partial^2}{\partial t^2} + \frac{1}{t}\frac{\partial}{\partial t} - \frac{(m\pm 1)}{t^2} + \frac{\partial^2}{\partial p^2} - \pi S^2 h\right](A_r \pm A_\varphi) = 0 \quad (8a)$$

with the boundary conditions

$$\partial(A_r \pm A_\varphi)/\partial n = 0. \quad (8b)$$

These equations are the same as the curling equation (7), except for the factor multiplying $-t^{-2}$, which is *larger* for $m \geqslant 2$ than for the curling. They, thus, come under the following mathematical theorem proved by Titchmarsh[6]:

Let the equation

$$[\nabla^2 + \lambda - q(x_1, x_2, \cdots)]\psi = 0$$

be defined in any region $E$, with the boundary conditions[7] $\partial\psi/\partial n = 0$. Then each of its eigenvalues, $\lambda$, is nondecreasing as $q$ is increased, i.e., when $q$ is replaced by any other function of space $Q(x_1, x_2, \cdots)$, provided $q \leqslant Q$ throughout $E$.

In particular, in our case, $-h$ for Eq. (8) cannot be smaller than the corresponding $-h$ for Eq. (7). Thus, for $m \geqslant 2$ no eigenvalue can be numerically smaller than for the curling, even when the transverse self-magnetostatic energy is dropped, even less so when this energy is retained.

It has, thus, been proved that for $m \neq 1$ in (6), the lowest eigenmode is the curling as given by (7). It should be noted that the proof did not use the prolate shape, although the cylindrical symmetry is essential. Thus, the proof applies to oblate as well as to prolate spheroids. It also covers special cases of it which were proved separately for the cylinder,[3] the unidirectional cylinder[8] and the sphere.[4] It also applies for the case when $K$ is a function of space and, thus, includes a recent model for nucleation around dislocations,[9] where a special case of this theorem was proved numerically.

Besides the curling, one is, thus, left with the case $m=1$. In the infinite cylinder[3] this yielded two modes of interest, namely, the coherent rotation and the buckling. We shall, therefore, give the name "buckling" in this case too, to the mode, orthogonal on the coherent

[6] E. C. Titchmarsh, *Eigenfunction Expansions Associated with Second Order Differential Equations* (Oxford University Press, London, 1958), Part II, pp. 88–90.

[7] The theorem is stated in Ref. 6 for boundary conditions $\psi=0$. However, its proof uses only the condition that $\psi\partial\psi/\partial n$ vanishes on the surface of $E$, and can, therefore, be used for either one of these boundary conditions.

[8] A. Aharoni, E. H. Frei, and S. Shtrikman, J. Appl. Phys. **30**, 1956 (1959).

[9] C. Abraham and A. Aharoni, Phys. Rev. **128**, 2496 (1962).

rotation, which yields the numerically smallest eigenvalue when $m=1$. This will be treated in the next part.

### 3. MAGNETIZATION BUCKLING

Let (6), with $m=1$, be substituted in (1), (2), and (3), changing over from the parameters $A_r, A_\varphi$ to

$$B_1=A_r+A_\varphi, \quad B_2=A_r-A_\varphi. \tag{9}$$

Let the cylindrical coordinate $t,p$, be transformed to prolate spheroidal coordinates[10] according to

$$t=(1-\eta^2)^{1/2}(\xi^2-1)^{1/2}(\xi_0^2-1)^{-1/2}, \tag{10a}$$

$$p=\xi\eta(\xi_0^2-1)^{-1/2}, \tag{10b}$$

where $\xi=\xi_0$ [which is a prolate spheroid with radii 1 and $\xi_0(\xi_0^2-1)^{-1/2}$, respectively, in the reduced $t,p$ coordinate system] represents the surface of the ferromagnetic particle. The equations inside the material, i.e., for $\xi\leqslant\xi_0$ are then

$$\{\nabla^2-4(\xi^2-\eta^2)(1-\eta^2)^{-1}(\xi^2-1)^{-1} + c^2(\xi^2-\eta^2)\}B_1+\pi M(L_1-L_2)V=0, \tag{11a}$$

$$\{\nabla^2+c^2(\xi^2-\eta^2)\}B_2-\pi M(L_1+L_2)V=0, \tag{11b}$$

$$\{\nabla^2-(\xi^2-\eta^2)(\xi^2-1)^{-1}(1-\eta^2)^{-1}\}V - M\{2L_1B_1+L_2(B_1+B_2)\}=0, \tag{11c}$$

where

$$\nabla^2=\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}(1-\eta^2)\frac{\partial}{\partial\eta}, \tag{12a}$$

$$L_1=(\xi^2-\eta^2)(1-\eta^2)^{-1/2}(\xi^2-1)^{-1/2}, \tag{12b}$$

$$L_2=(\xi^2-1)^{1/2}(1-\eta^2)^{1/2}(\xi\,\partial/\partial\xi-\eta\,\partial/\partial\eta), \tag{12c}$$

$$c^2=-\pi S^2h(\xi_0^2-1)^{-1}, \tag{13a}$$

$$M=S(\xi_0^2-1)^{-1/2}. \tag{13b}$$

In the same way, one obtains

$$\{\nabla^2-(\xi^2-\eta^2)(\xi^2-1)^{-1}(1-\eta^2)^{-1}\}V=0, \quad \xi>\xi_0 \tag{14}$$

and on the boundary, $\xi=\xi_0$:

$$\partial B_1/\partial\xi=\partial B_2/\partial\xi=0 \tag{15a}$$

$$V_{\text{in}}=V_{\text{out}} \tag{15b}$$

$$\partial V_{\text{in}}/\partial\xi-\partial V_{\text{out}}/\partial\xi = M(B_1+B_2)\xi_0(\xi_0^2-1)^{-1/2}(1-\eta^2)^{1/2}. \tag{15c}$$

One solution of this set of equations can be written immediately, namely,

$$B_1=0, \quad B_2=\text{const}, \quad V_{\text{in}}=(c^2B_2/2\pi M)(1-\eta^2)^{1/2}(\xi^2-1)^{1/2}. \tag{16a}$$

This is the coherent rotation. It yields, after using the boundary conditions, the eigenvalue

$$-h_n=\xi_0\{\xi_0^{-1/2}(\xi_0^2-1)\ln[(\xi_0+1)/(\xi_0-1)]\}. \tag{16b}$$

[10] Carson Flammer, *Spheroidal Wave Functions* (Stanford University Press, Stanford, California, 1957).

Being an eigenmode, it should be orthogonal on all the other eigenmodes of the set of Eqs. (11), (14), and (15). The buckling mode, according to our definition, is thus the solution of this set of equations, which is orthogonal to (16), and yields the smallest eigenvalue $c$.

It is always possible to expand the solution as a series in a complete orthogonal set of functions. We chose these functions to be the eigenmodes of the *homogeneous* part of (11a) and (11b) with the boundary conditions (15a). This implies[11]

$$B_1=\sum_{n=2}^{\infty}\sum_{k=1}^{\infty} a_{kn}S_{2n}(c_{kn}^{(2)},\eta)R_{2n}^{(1)}(c_{kn}^{(2)},\xi) \tag{17a}$$

$$B_2=\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} b_{kn}S_{0n}(c_{kn}^{(0)},\eta)R_{0n}^{(1)}(c_{kn}^{(0)},\xi) + \sum_{k=2}^{\infty} b_{k0}S_{00}(c_{k0}^{(0)},\eta)R_{00}^{(1)}(c_{k0}^{(0)},\xi). \tag{17b}$$

Here $c_{kn}^{(m)}$ are the zeros of the derivative of $R_{mn}^{(1)}$ in increasing order, i.e.,

$$[dR_{mn}^{(1)}(c_{kn}^{(m)},\xi)/d\xi]_{\xi=\xi_0}=0, \quad c_{k+1,n}^{(m)}>c_{kn}^{(m)}. \tag{18}$$

It should be noted that in the second sum of (17b), the summation starts with $k=2$, rather than $k=1$. This means that the term with $c_{10}^{(0)}$ has been excluded from the expansion. However, $c_{10}^{(0)}=0$, and

$$R_{00}^{(1)}(0,\xi)=S_{00}(0,\eta)=1,$$

which means that the term missing in (17b) is a constant. Now, $B_2=\text{const}$ is the coherent rotation, and as mentioned before, is orthogonal on all the other eigenfunctions, and can thus be treated separately. The expansion (17) is, therefore, the most general form for the solution of our set of equations, which does not include the coherent rotation mode.

In principle, (17), (14), and (15) determine uniquely the potential $V$, both inside and outside of the spheroid. When substituted in (12), one should, therefore, obtain a set of *algebraic* linear equations in the parameters $a_{kn}$, $b_{kn}$, then obtain an equation for the eigenvalues by equating to zero the determinant of the coefficients of $a_{kn}$ and $b_{kn}$. However, this is cumbersome. Instead, we shall get just a lower bound for the buckling eigenvalue, by neglecting the transverse magnetostatic self-energy. It is readily seen from the derivation of the equations, that neglecting this energy means writing $V=0$ in (11a), (11b) and disregarding (11c), (14), (15b), and (15c). In this case the determinant is diagonalized, and one obtains for the eigenvalues

$$c=c_{kn}^{(0)} \quad \text{or} \quad c=c_{kn}^{(2)},$$

whereas the curling eigenvalue was[4] in the present notation,

$$c=c_{11}^{(1)}.$$

[11] The notations used here are according to Ref. 10.

According to the theorem of Titchmarsh, mentioned in Sec. 2,

$$\text{smallest } c_{kn}^{(2)} \geqslant \text{smallest } c_{kn}^{(1)} \geqslant \text{smallest } c_{kn}^{(0)}, \quad (19)$$

so that one need consider only the smallest of the eigenvalues

$$c = c_{1n}^{(0)}, \quad n \geqslant 1, \quad \text{or} \quad c_{20}^{(0)} \quad (20)$$

[since $c_{10}^{(0)}$ has been excluded from (17), before neglecting the magnetostatic energy].

In the analogous treatment of the sphere,[4] all the eigenvalues analogous to (20) were not smaller than the curling eigenvalue [this does not contradict (19), since the smallest eigenvalue has been removed]. For the sphere, therefore, the buckling eigenvalue is larger than the curling one, and can thus be ignored. This is not the case for prolate spheroid, where (20) yields eigenvalues smaller than those of the curling. Using tabulated spheroidal functions, and their expansions[10] the smallest eigenvalue, $c_{11}^{(0)}$, was computed. It is plotted in Fig. 1 as a function of the elongation, $m = \xi_0(\xi_0^2 - 1)^{-1/2}$, together with the curling eigenvalue for comparison.

It is seen from Fig. 1 that the buckling eigenvalues are always smaller than the curling ones (except for the sphere, $m=1$, where they are equal). However, this is only a lower bound, and the actual buckling eigenvalue should certainly become larger than the curling ones, at least for large enough $S$. It is even possible that for certain elongations the buckling eigenvalue would turn out to be larger than the curling one for every $S$, as is the case for the sphere. Near the sphere, i.e., for not too elongated particles, the difference between the two curves of Fig. 1 does not justify going into detailed calculations of the complicated buckling mode. When $1/m \rightarrow 0$, i.e., for very elongated particles, the difference between the curves becomes much larger. However, in this region the behavior cannot be essentially different from that of the infinite cylinder, where it is known[3] that the buckling eigenvalues are very close to coherent rotation, so that again detailed calculation does not seem necessary.

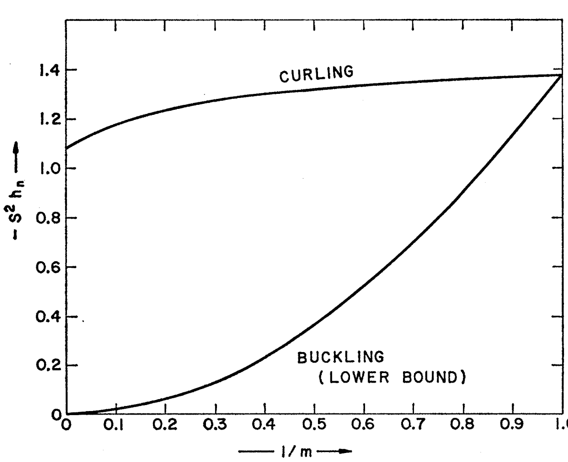


FIG. 1. The reduced nucleation field $h_n$ in terms of the reduced radius $S$, both defined in (5), as a function of the reciprocal of elongation, $1/m$, for a prolate spheroid.

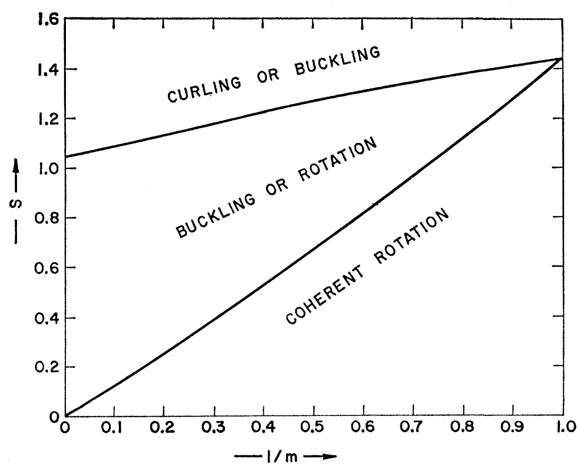


FIG. 2. Upper and lower bounds of the critical reduced size $S$, defined by Eq. (5), for coherent rotation, as functions of the reciprocal of elongation, $1/m$, in a prolate spheroid. The possible modes in the three regions separated by the curves, are marked on the figure.

The simplest mode to treat theoretically after nucleation is the coherent rotation, and has, therefore, been used in many calculations. From Fig. 1 we can get upper and lower bounds for the size for which these calculations are valid, by equating each of the modes to the coherent rotation eigenvalues. The results are plotted in Fig. 2. In this figure the lower curve represents the size in which coherent rotation nucleation fields just equals that of the lower bound for buckling. Below this curve, therefore, the coherent rotation is the lowest mode. The upper curve represents the radius at which nucleation by coherent rotation equals that by curling. Above this curve, therefore, coherent rotation can no more take place. Again, the upper and lower bounds are reasonably close together to give a good approximation for the critical size, especially since the two extremes ($m=1$ and $m=\infty$) on the *lower* curve are exact. In particular, it is seen that for the range of superparamagnetism[12] the use of coherent rotation is justified, except for the very elongated particles.

Above the upper curve in Fig. 2, either buckling or curling can take place. However, for the infinite cylinder, the turn over from buckling to curling is only very slightly above this curve, and for the sphere no buckling takes place at all. It can, thus, be assumed that just above the upper curve, curling gives the lowest nucleation, thus restricting the buckling to part of the region between the curves, which can be regarded as an insignificant transition region between curling and coherent rotation.

[12] C. P. Bean and J. D. Livingston, J. Appl. Phys. **30**, 120S (1959).

## 4. REMARKS ON OBLATE SPHEROID

The calculations of Sec. 2 apply to oblate as well as to prolate spheroids. In this case too one has therefore just the curling, represented by (7), the coherent rotation which is known, and the mode analogous to the buckling treated in Sec. 3, which can be readily represented by transforming (11), (14), and (15) to oblate spheroidal coordinates. The fact that the curves in Fig. 1 cut at the sphere shows that for oblate spheroids the mode shown in Fig. 1 is higher than the curling, since it seems unlikely that the curves would cross again. However, it is possible that one of the other eigenvalues, which is larger for prolate spheroids, would also cross these curves at the sphere, and would thus become smaller than curling for oblate spheroids, so that there still exists the possibility of a third mode. For lack of adequate tabulation of the oblate functions, this possibility could not be checked.

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# Size Effects in the Resistivity of Indium Wires at 4.2°K

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Measurements are reported of the dependence of the resistance (at 4.2°K) of high-purity polycrystalline indium wires on the wire diameter. Data, which were taken on recrystallized wires extruded through dies of various sizes, and also on a single extruded wire gradually reduced in diameter by etching, are compared with those of Olsen. It is pointed out that any variation of the bulk electron free path over the Fermi surface must be taken into account in the analysis of size effect data on wires unless they are extremely small in diameter. A calculation of the size effect at 0°K in monocrystalline wires and in "unidimensionally" polycrystalline wires having a diameter large compared to the mean free path is made for an arbitrary Fermi surface and free path anisotropy. The result of the calculation for the polycrystalline case, which is limited to metals having isotropic bulk conductivities, is similar to the Fuchs-Dingle result for the isotropic case except that the effective resistivity is much more strongly size dependent when a large mean free path anisotropy exists. It is concluded on the basis of this derivation that the size effect data on indium wires and anomalous skin effect data can be reconciled if a large anisotropy in the mean free path exists.

## INTRODUCTION

THE dependence of the resistance of circular wires on diameter has been studied theoretically[1–7] and also experimentally[6,8,9] for several metals. Experimental data of this type have frequently been analyzed by means of the Nordheim-Fuchs-Dingle[1,2] formula,

$$\rho_{\mathrm{eff}}=\rho_b+\alpha\rho_b l/d\,, \tag{1}$$

which (assuming diffuse surface scattering) expresses the effective resistivity, $\rho_{\mathrm{eff}}$, of the wire in terms of the bulk resistivity, $\rho_b$, the mean free path, $l$, and the diameter, $d$. $\alpha$ is a dimensionless function of $l/d$ which is unity in the Nordheim formula and varies from 0.75 to 1 as $l/d$ goes from zero to infinity in the Fuchs-Dingle formulation. Equation (1) is often used to calculate the product $\rho_b l$ and the mean free path from size effect data.[10] The value of $\rho_b l$ obtained in this way is usually considerably larger than the value derived from anomalous skin effect data on polycrystalline samples.

The purpose of this paper is to report measurements of the size effect in polycrystalline indium wires and to point out that Eq. (1) is not applicable to these data. An equation similar to (1) [Eq. (1c)], which is believed to be valid for the residual resistivity of thick "one-dimensionally" polycrystalline wires of metals having arbitrary Fermi surfaces and an arbitrary dependence of the free path $l(k_f)$ (averaged over all final wave vectors), on the initial wave vector is derived. This formula is probably more appropriate to the case of "annealed" polycrystalline wires than is Eq. (1).

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